**R15 INTERPRET THE GRADIENT AT A POINT ON A CURVE AS THE INSTANTANEOUS RATE OF CHANGE; APPLY THE CONCEPTS OF AVERAGE AND INSTANTANEOUS RATE OF CHANGE (GRADIENTS OF CHORDS AND TANGENTS) IN NUMERICAL, ALGEBRAIC AND GRAPHICAL CONTEXTS (THIS DOES NOT INCLUDE CALCULUS) (higher tier)**

**GRADIENTS OF CURVES**

Graphs are an important part of determining a rate of change. At this level, you should be able to find the gradient to a curve.

**C**

The curve, shown, varies in steepness.

The section between B and C is steeper than that between A and B.

By definition gradient is a measure of steepness, the gradient of the curve changes throughout.

**B**

**A**

As we move along the curve from A to B the gradient increases.

If, at B, we continue straight ahead (along the dotted line) the gradient stops increasing and has the same value as at B.

The gradient of the curve at B is then the gradient of the dotted line.

**C**

**B**

**A**

**C**

**B**

**A**

The dotted line at B may be extended backwards as shown here.

The dotted line becomes a solid line.

This line **touches** the curve at B and is called the **tangent** at B.

In summary,

The gradient of a curve at a point is the gradient of the tangent to the curve at that point.

How do we find the gradient of this curve at the point P?

*x*

*y*

20

40

60

10

0

20

30

40

**P**

tangent

38 − 16 = 22

40 − 10 = 30

To find the gradient at point P:

* Draw a tangent to the point P
* Use the tangent (hypotenuse) to complete a right-angled triangle

Hence,



**EXAMPLE 1**

Asha poured liquid into a container.

She recorded her results and then drew a graph as shown.

Time (seconds)

Depth (cm)

10

20

30

20

0

40

60

80

100

38 − 16 = 22

(a) Work out the gradient of this curve when *t* = 60 seconds.

(b) Interpret your answer to part (a).

First draw a tangent at *t* = 60 seconds (shown in **red** on the graph).

Then draw a right-angled triangle (shown in **blue** and **green** on the diagram).

Try to draw the triangle so that it goes through easy points to read on the graph.

(a)

Change in depth (*y*) (in blue) = 22 − 3 = 19 Read the *y*-coordinates on your triangle

Change in time (*x*) (in green) = 86 − 34 = 52 Read the *x-*coordinates on your triangle



(b) When *t* = 60 seconds the depth of the water Vertical distance = cm

is increasing at a rate of 0.365 cm per second Horizontal distance = seconds  
 cm/sec = cm per second

Depth is increasing as positive gradient

**EXAMPLE 2**

In an experiment, Anjali heated a liquid to 60 °C then allowed it to cool.

The graph shows this information.

Time (minutes)

Temperature (°C)

20

40

60

10

0

20

30

40

50

(a) Work out the average rate of decrease of temperature between *t* = 0 and *t* = 50.

(b) Work out the rate of decrease of temperature at *t* = 20. State the units.

(a)

Draw a line from *t*= 0 to *t* = 50 and then complete a triangle.

Time (minutes)

Temperature (°C)

20

40

60

10

0

20

30

40

50

50 − 0 = 50

60 − 4 = 56

Gradient is negative as slopes downwards

= – 1.12°C per minute

Always put the units with your answer. 56 °C ÷ 50 minutes = 1.12 °C per minute

(b)

Time (minutes)

Temperature (°C)

20

400

60

10

0

20

30

40

50

Draw a tangent to the curve at the point *t* = 20

42 − 6 = 36

40 − 3 = 37

 = – 0.973 °C per minute

Gradient is negative as slopes downwards.

Always put the units with your answer. −36 °C ÷ 37 minutes = − 0.973 °C per minute

**EXAMPLE 3**

This graph shows the voltage across a phone battery as it charges from empty.

25

Voltage

(V)

1

0

5

10

15

20

2

3

4

5

6

Time (minutes)

30

35

40

45

(a) Work out the average rate of increase of voltage between *t* = 0 and *t* = 45

Ravina wants to stop charging her phone when the rate of increase of voltage drops

to this average level.

(b) After how long should Amir stop charging his phone?

You must show how you got your answer.

(a)

25

Voltage

(V)

1

0

5

10

15

20

2

3

4

5

6

Time (minutes)

30

35

40

45

5.8 − 0 = 5.8

Draw a triangle using *t* = 0 and *t* = 45 as the base

45 − 0 = 45

gradient = volts per minute

Always put the units with your answer. 5.8 volts ÷ 45 minutes = 0.129 volts per minute

(b) Draw a tangent to the curve which is parallel to the average rate line.

This is shown in blue on the diagram.

25

Voltage

(V)

1

0

5

10

15

20

2

3

4

5

6

Time (minutes)

30

35

40

45

Amir should stop charging The tangent cuts the curve when *t* = 21 minutes

his phone after 21minutes.

**EXERCISE:**

1. The diagram shows parts of the graphs of *y* = f(*x*) and *y* = g(*x*).



(a) Write down the value of *x* where the gradient of the curve *y* = g(*x*) is zero.

(b) Calculate an estimate for the gradient of the curve *y* = f(*x*) at the point on the curve

where *x* = 4

2. A fish bowl is being filled with water.

The graph shows how the diameter of the surface of the water changes with time.

25

Diameter (cm)

10

0

5

10

15

20

20

30

40

50

60

Time (seconds)

30

35

40

45

50

(a) Work out the gradient when the time is 10 seconds.

(b) Give an interpretation of the gradient.

3. The graph gives information about the variation in the temperature, in °C, of an amount

of water that is allowed to cool from 80 °C.



(a) Work out the rate of decrease of temperature at *t* = 400

(b) Work out the average rate of decrease of the temperature of the water between

*t* = 0 and *t* = 800.

The instantaneous rate of decrease of the temperature of the water at time *T* seconds is equal to the average rate of decrease of the temperature of the water between *t* = 0 and *t* = 800

(c) Find an estimate for the value of *T.*

You must show how you got your answer.

4. A tank was emptied and the depth of water was recorded.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Time (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Depth (metres) | 30 | 29.5 | 29 | 28 | 27 | 26 | 24.5 | 22.5 | 19.5 | 15 | 9 |

(a) Plot the graph of depth against time.

(b) Work out the average rate of decrease of the depth of the water between *t* = 0 and *t* = 10

(c) Work out the gradient at *t* = 7

(d) Give an interpretation of part (c).

5. An experiment was conducted. Jimmy took the following measurements for the length of the wire, L, and the tension, T.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Length (cm) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Tension (N) | 16 | 30 | 39 | 45 | 50 | 54 | 56 | 58 | 59 | 60 | 61 |

(a) Plot the graph of tension against length.

(b) Work out the gradient when the length of the wire is 55 cm.

(c) Explain what this gradient tells you.

**ANSWERS**

**Exercise**

1. (a) 3.0 (b) 2.86

2. (a) 1.4 (b) The diameter is increasing by 1.4 cm per second.

3. (a) 0.056 (b) 0.06 (c) 300

Depth (metres)

0

2

4

10

20

30

Time (minutes)

6

8

10

4. (a)

(b) −2.1 (c) −2.5

(d) The depth of water decreases by 2.5 cm per minute

5. (a)

Tension (Newtons)

0

20

40

20

40

60

Length (cm)

60

80

100

(b) 0.34 (c) The tension increases by 0.34 Newtons per cm